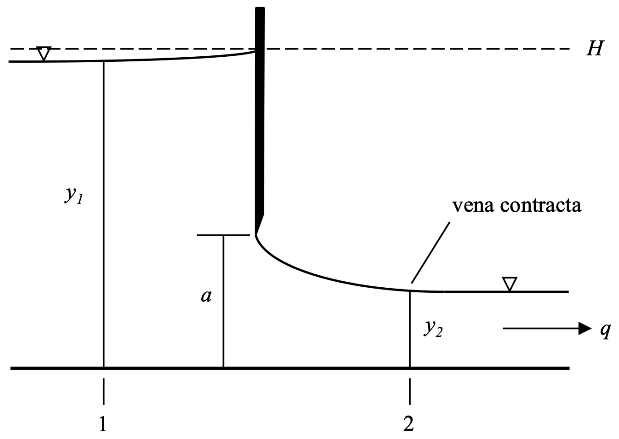
Example 1 - flow discharge through an unsubmerged underflow gate



A 4.3 m wide canal has an upstream depth of 2.7 m. A 1.5 m wide sluice gate is opened 45 cm. After contracting, the downstream depth is 27.5 cm.

What is the discharge exiting through the gate?

$b = 1.5\ m$ (this is the width of the gate, not to be confused with the width of the channel. The Q equation for gates refers to this)

$Q = ? \longrightarrow Q = b \cdot C_d\, a \sqrt{2 g y_1}$ with $C_d = \dfrac{C_c}{\sqrt{\dfrac{a C_c}{y_1} + 1}}$

$y_1 = 2.7\ m$

$y_2 = 0.275\ m$

$a = 0.45\ m$

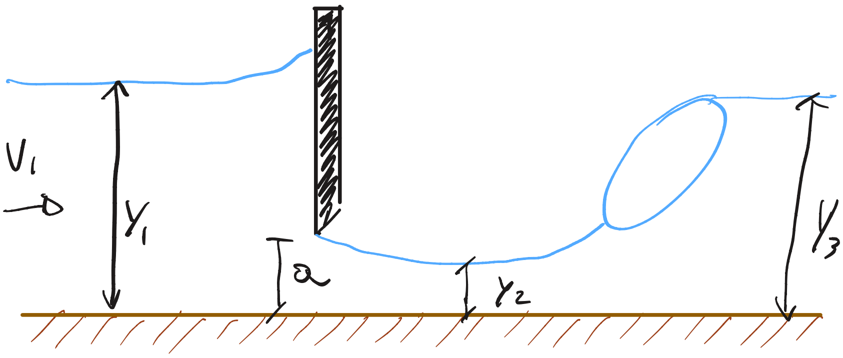
$\Rightarrow C_c = \dfrac{y_2}{a} = \dfrac{0.275\ m}{0.45\ m} = 0.611$

$$C_d = \frac{0.611}{\sqrt{\dfrac{0.45 m \cdot 0.611}{2.7\ m} + 1}} = 0.582$$

$$\Rightarrow Q = (1.5\ m)(0.582)(0.45\ m)\sqrt{2(9.81\ m/s)(2.7\ m)}$$

$$= 2.86\ m^3/s \quad ✓$$

Example 2 - Preventing the gate to be downed



A 5 m wide canal with 5 m wide gate with a flow discharge of 25 m3/s. The slope is 0.003 and Manning coefficient is 0.02
For a Cc=0.6,

- how much can the gate open without drowning the hydraulic jump (and so the gate flow)?
- What is the depth behind the gate?

$Q = 25\ m/s$

$S_0 = 0.003$

$n = 0.02$

$b = 5\ m$

$C_c = 0.6$

(1) We calculate the conditions downstream (NORMAL & CRITICAL DEPTH) to see if the HJ would occur in the first place:

- NORMAL DEPTH → $Q = \frac{1}{n} A R_H^{2/3} S^{1/2}$

$Y_n = Y_3$

$$25\ m^3/s = \frac{1}{0.02}(5_m \cdot Y_n)\left(\frac{5m \cdot Y_n}{5m + 2Y_n}\right)^{2/3} \cdot (0.003)^{1/2}$$

$$Y_n = 1.78\ m$$

- CRITICAL DEPTH → $Y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{25^2}{5^2 \cdot 9.81}\right)^{1/3} = 1.36\ m$

Since $Y_n > Y_c \Rightarrow$ DOWNSTREAM channel is in SUBCRITICAL, so there is going to be a HJ somewhere:

Depending on the aperture of the gate, the HJ will be pushed more or less downstream. Specifically, the more we open the gate, the less inertia the outflow has and so the closer the HJ will get to the gate (pushed by the downstream momentum).

So to know the maximum opening possible, we calculate the opening such that the vena contracts (the outflow depth) is the conjugated depth of the HJ.

CONJ DEPTH EQ $\Rightarrow \dfrac{y_2}{y_3} = \dfrac{1}{2}\left[-1+\sqrt{1+8F_{2_3}^2}\right]$

$F_{2_3}^2 = \dfrac{V_3^2}{y_3 g} = \dfrac{Q^2}{b^2 y_3^2 y_3 g} = \dfrac{25^2}{5^2\, 1.78^3\, 9.81} = 0.45$

$y_2 = \dfrac{1.78\,m}{2}\left[-1+\sqrt{1+8\cdot(0.45)}\right] = 1.02\ m$

↳ this must be the outflow depth ("vena contracta")

⇓

$y_2 = a \cdot C_c$

$a = \dfrac{y_2}{C_c} = \dfrac{1.02\,m}{0.61} = 1.675\ m$

⇒ how much is y_1?

$Q = b\, C_d \sqrt{2gy_1}$ with $C_d = \dfrac{C_c}{\sqrt{\frac{aC_c}{y_1}+1}}$

$= \dfrac{b\, C_c}{\sqrt{\frac{aC_c}{y_1}+1}}\sqrt{2gy_1} \Rightarrow 25\,\frac{m^3}{s} = 5\,m \cdot \dfrac{(0.6)}{\sqrt{\frac{1.02\,m}{y_1}+1}} \cdot \sqrt{2\cdot(9.81\,m/s^2)\,y_1}$

using a solver or Try & Error

✓ $y_1 = 1.883\ m$